

HOPF AND TORUS BIFURCATIONS IN STOCHASTIC SYSTEMS IN MATHEMATICAL POPULATION BIOLOGY

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ABSTRACT

The classical Rosenzweig-MacArthur model shows the transition from a stable fixed point to a limit cycle via a Hopf bifurcation[1]. However, the Holling type II response function, which in this model allows a Hopf bifurcation due to the upcoming cubic nonlinearity[1], is not directly related to a transition from one to another population class which would allow a stochastic version straight away. Instead, a time scale separation argument leads from a more complex model to the simple 2 dimensional Rosenzweig-MacArthur model, via additional classes of food handling and predators searching for prey. This extended model allows a stochastic generalization with the stochastic version of a Hopf bifurcation, and ultimately also with additional seasonality allowing a torus bifurcation[1]. Routes to chaos not only via Feigenbaum period doubling but also via torus bifurcations seem more widely present in population biology, and were for example found in extended multi-strain epidemiological models on dengue fever. To understand such dynamical scenarios better also under noise the present low dimensional system can serve as a good study case.

References

- [1] Y.A. KUZNETSOV, *Elements of applied bifurcation theory*, Springer-Verlag, New York, third edition 2010.