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COMPARISON BETWEEN VARIABLE AND CONSTANT EFFORT SUSTAINABLE HARVESTING POLICIES FOR LOGISTIC RANDOM ENVIRONMENTAL MODELS

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ABSTRACT

To describe the growth of a harvested population (in fisheries, forestry, etc.) when the environment is subjected to random fluctuations, one can use Stochastic Differential Equation (SDE) models (see, for example, [1] and [2]). Here we consider a logistic type average natural growth to which we subtract a harvesting yield term of the form $h(t) = qE(t)X(t)$, where $q > 0$ is the catchability coefficient, $E(t) \geq 0$ is the harvesting effort of the adopted harvesting policy and $X(t)$ is the population size at time t.

There is previous work on the optimal design of the harvesting policy with the purpose of maximizing the accumulated profit (discounted by a depreciation rate) over a finite time horizon (see, for example, [3]). The optimal policies require variable harvesting efforts which, under certain conditions, are even of bang-bang type (consisting in constantly alternating between short periods of harvesting and no-harvesting, according to the randomly varying population size). This type of policies could be applicable to financial assets, which can be evaluated and traded almost continuously, but they are not applicable to harvesting. In fact, evaluation of population size is difficult, expensive and time consuming, and the logistics of harvesting are not compatible (both from the practical and the social implications points of view) with very frequent randomly determined changes in harvesting effort.

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An alternative methodology was proposed (see, for instance, [4] and [5]), based on sustainable and applicable fishing policies that also lead to sustainability of the population and a stationary distribution of the population size. We determine the constant harvesting effort policy that optimizes the (also constant) expected sustainable profit per unit time; let $\hat{E}$ be the optimal harvesting effort and $\hat{P}$ the optimal such profit rate.
Then, using Monte Carlo simulations, we compare the two methodologies, namely by comparing $\hat{E}$ and $\hat{P}$ of the alternative methodology with the optimal time-varying effort $\hat{E}(t)$ and the optimal time-varying expected profit per unit time $\hat{P}(t)$ of the first methodology. We can then check what we lose profitwise by using the alternative sustainable and applicable policy instead of the absolute optimal inapplicable policy.

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